SRI BHARATHI
ENGINEERING COLLEGE FOR WOMEN

(Approved by AICTE, New Delhi and Affiliated to Anna University, Chennai)
Kaikkurichi, Pudukkottai -622 303

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NAAC DOCUMENTS

Quality Indicator Frame Work

Criterion – 2

Teaching-Learning and Evaluation

Submitted by

IQAC

Internal Quality Assurance Cell

Key Indicator- 2.3. Teaching- Learning Process (40)

2021-2022

SCIENCE AND HUMANITIES

PROBLEM SOLVING

Activity	Number of Students Attended	Page No.
Tutorial	50	3
TOTAL STUDENTS ATTENDED	50	-

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Criteria 2	Teaching-Learning and Evaluation	350

Key Indicator- 2.3. Teaching- Learning Process (40)

2021-2022

SCIENCE AND HUMANITIES

PROBLEM SOLVING

TUTORIAL

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DEPARTMENT OF SCIENCE AND HUMANITIES

ACEDEMIC YEAR (2021-2022)-ODD SEMESTER

PROBLEM SOLVING METHOD

SL.NO	REG.NO	NAME	YEAR/SEC	LEARNING METHOD
1.	912621104001	ABINAYA.K	I/A	PROBLEM SOLVING METHOD-TUTORIAL MA3151-MATRICES AND CALCULUS
2.	912621104002	AMEERA.N	I/A	
3.	912621104003	ANJUGAM.C	I/A	
4.	912621104004	ARUNDATHI.S	I/A	
5.	912621104005	ASHIKA.B	I/A	
6.	912621104006	DIVYA.T	I/A	
7.	912621104007	ELACKIYA.G	I/A	
8.	912621104008	GAYATHRI.K	I/A	
9.	912621104009	GEETHA.M	I/A	
10.	912621104010	HARSHITHA.P	I/A	
11.	912621104011	ISHWARYA.S	I/A	
12.	912621104012	JANANI.R	I/A	
13.	912621104013	KANDALAKSHMI.A	I/A	
14.	912621104015	LAVANYA.S	I/A	
15.	912621104016	MAHASREE.P	I/A	
16.	912621104018	PRIYA.M	I/A	
17.	912621104019	RABIKA.R	I/A	
18.	912621104020	RISVANA BEGAM.S	I/A	
19.	912621104021	SAHEENA BEGAM.A	I/A	
20.	912621104022	SASIPRIYA.R	I/A	
21.	912621104023	SHAMIMA.P	I/A	
22.	912621104024	SHEERA BANU.A	I/A	
23.	912621104025	SIVAJOTHIKA.S	I/A	
24.	912621104026	SIVAPRIYA.R	I/A	
25.	912621104027	SUBADHARSINI.S	I/A	
26.	912621104028	SUBIKSHA.S	I/A	
27.	912621104029	VINITHA.K	I/A	
28.	912621104030	VISALATCHI.S	I/A	
29.	912621104031	YOGESHWARI.S	I/A	
30.	912621103001	AKILA.G	I/B	
31.	912621103002	GAYATHRI.G	I/B	
32.	912621103003	JAYABHARATHI.R	I/B	
33.	912621103004	JAYA MANOHARI.B	I/B	
34.	912621103005	PRIYADHARSHINI.A	I/B	
35.	912621103006	RABIA BANU.M	I/B	
36.	912621103007	SHERLIN KAVYA.B	I/B	
37.	912621106001	AMRIN. M	I/B	
38.	912621106002	BHUVANESWARI.C	I/B	
39.	912621106003	DHANYASHREE.A	I/B	
40.	912621106004	KALAIVANI.R	I/B	
41.	912621106005	KAVIYA.K	I/B	
42.	912621106006	KEERTHANA.V	I/B	
43.	912621106007	PAVITHRA.P	I/B	

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44.	912621106008	RAJESHWARI.R	I/B	PROBLEM SOLVING METHOD-TUTORIAL MA3151-MATRICES AND CALCULUS
45.	912621106009	SUBALAKSHMI.M	I/B	
46.	912621106010	SUGUNA.C	I/B	
47.	912621105001	GOKULA PRAVEENA.A	I/B	
48.	912621105002	RAFEEQA.N	I/B	
49.	912621105003	RAJESWARI. A	I/B	
50.	912621105004	SUMITHRA.S	I/B	

Name and signature of the faculty Incharge

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ACADEMIC YEAR 2021 – 2022 (ODD SEMESTER)
DEPARTMENT OF SCIENCE AND HUMANITIES

Tutorial Question Paper

Tutorial – 01			Date of Issue:	24.08.2021	Marks	40
Course code	MA3151	Course Title	MATRICES AND CALCULUS			
Year	I	Semester/Section	I / B	Date of Submission:	26.08.2022	

Q.No	Questions	CO
1	Find the Eigen values and Eigenvectors of the matrix $A = \begin{pmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{pmatrix}$	C102.1
2	Find the Eigen values and Eigenvectors of the matrix $A = \begin{pmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{pmatrix}$	C102.1
3	Using Cayley-Hamilton theorem find $A^{-1}, if\ A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$	C102.1

N. Vithya [N. Vithya]
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DEPARTMENT OF SCIENCE AND HUMANITIES

Tutorial Answer Sheet

Name of the Student : M. Amrin

AU Register Number: 912621106001

Tutorial – 01				Date of Issue:	02.12.2021	Marks	10
Course code	MA3152	Course Title	MATRICES AND CALCULUS				
Year	I	Semester/Section	I/B	Date of Submission:	06.12.2021		

Q.No	Questions	CO
1	Find the Eigen values and Eigenvectors of the matrix $A = \begin{pmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{pmatrix}$	C102.1
2	Find the Eigen values and Eigenvectors of the matrix $A = \begin{pmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{pmatrix}$	C102.1
3	Using Cayley-Hamilton theorem find A^4 and $A^{-1}, if\ A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$	C102.1

Mark Allocation

Rubrics	Marks Allocated	Marks obtained
Problem solving approach	6	5
Correctness of Answer	2	2
Timely submission	2	2
Total marks	10	09

(N. Vithya)
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1. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$

Sol:

Step 1: To characteristic equation

$$A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$$

characteristic equation

$$\lambda^3 - C_1\lambda^2 + C_2\lambda - C_3 = 0$$

C_1 = Sum of principal diagonal element

$= 7 + 6 + 5$

$C_1 = 18$

C_2 = Sum of minors of principal diagonal element

$$= \begin{vmatrix} 6 & -2 \\ -2 & 5 \end{vmatrix} + \begin{vmatrix} 7 & 0 \\ 0 & 5 \end{vmatrix} + \begin{vmatrix} 7 & -2 \\ -2 & 6 \end{vmatrix}$$

$= (30-4) + (35-0) + (42-4)$

$= 26 + 35 + 38$

$C_2 = 99$

$C_3 = |A|$

$$\begin{vmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{vmatrix}$$

$= 7(30-4) + 2(-10+0) + 0(4-0)$

$= 7(26) + 2(-10)$

$= 182 - 20$

$= 162$

$\therefore$ characteristic equation is

$$\lambda^3 - C_1\lambda^2 + C_2\lambda - C_3 = 0$$

$$\lambda^3 - 18\lambda^2 + 99\lambda - 162 = 0$$

Step 2 : To solve the characteristic equation

$$\lambda^3 - 18\lambda^2 + 99\lambda - 162 = 0$$

by trial and error method we can find one root.

(put $\lambda = 1, -1, 2, -2, 3$)

When $\lambda = 3$ the equation satisfied

$$(3)^3 - 18(3)^2 + 99(3) - 162 = 0$$

$$27 - 162 + 297 - 162 = 0$$

$$324 - 324 = 0$$

$$0 = 0$$

$\therefore \lambda = 3$ is one root of this equation using synthetic division

3	1	-18	99	-162
	0	3	-45	162
	1	-15	54	0

$$\lambda^2 - 15\lambda + 54 = 0$$

$$(\lambda - 9)(\lambda - 6) = 0$$

$$\lambda = 9, \lambda = 6$$

$\therefore$ Eigen values 3, 6, 9

$$\begin{bmatrix} -2 & -2 & 0 \\ -2 & -3 & -2 \\ 0 & -2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2x_1 - 2x_2 + 0x_3 = 0$

$-2x_1 - 3x_2 - 2x_3 = 0$

$0x_1 - 2x_2 - 4x_3 = 0$

From (i) & (ii) using by cross multiplication

$$\begin{matrix} -2 & 0 & -2 & -2 \\ -3 & -2 & -2 & -3 \end{matrix}$$

$$\frac{x_1}{4-0} = \frac{x_2}{0-4} = \frac{x_3}{6-4}$$

$\therefore$ Eigen vector $x_3 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

RESULT:

Eigen values 3, 6, 9

Eigen vectors $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

2) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$

Sol:

Step 1: To find characteristic equation

$$A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$$

characteristic equation $\lambda^3 - C_1\lambda^2 + C_2\lambda - C_3 = 0$

C_1 = (Sum of principal diagonal elements)

$= 11 - 2 - 6$

$= 11 - 8$

$= 3$

C_2 = (Sum of minors of principal diagonal element)

$= \begin{vmatrix} -2 & -5 \\ -4 & -6 \end{vmatrix} + \begin{vmatrix} 11 & -7 \\ 10 & -6 \end{vmatrix} + \begin{vmatrix} 11 & -4 \\ 7 & -2 \end{vmatrix}$

$= (12 - 20) + (-66 + 70) + (-22 + 28)$

$= -8 + 4 + 6$

$= 2$

$C_3 = |A|$

$= \begin{vmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{vmatrix}$

$= 11(12 - 20) + 4(-42 + 50) - 7(-28 + 20)$

$= -88 + 32 + 56 = -88 + 88$

$= 0$

∴ characteristic equation

$$\lambda^3 - C_1\lambda^2 + C_2\lambda - C_3 = 0$$

$$\lambda^3 - 3\lambda^2 + 2\lambda - 0 = 0$$

$$\lambda^3 - 3\lambda^2 + 2\lambda = 0$$

Step 2: Solve the characteristic equation to get eigen values

$$\lambda^3 - 3\lambda^2 + 2\lambda = 0$$

$$\lambda(\lambda^2 - 3\lambda + 2) = 0$$

$$\lambda = 0$$

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$\therefore$ Eigen vector $X_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

case 2 : When $\lambda = 1$ put in equ (1)

$$\begin{bmatrix} 11-1 & -4 & -7 \\ 7 & -2-1 & -5 \\ 10 & -4 & -6-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 10 & -4 & -7 \\ 7 & -3 & -5 \\ 10 & -4 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$10x_1 - 4x_2 - 7x_3 = 0 \longrightarrow (1)$$

$$7x_1 - 3x_2 - 5x_3 = 0 \longrightarrow (2)$$

$$10x_1 - 4x_2 - 7x_3 = 0 \longrightarrow (3)$$

consider (i) & ii) solve using cross multiplication

$$\begin{matrix} -4 & -7 & 10 & -4 \\ -3 & -5 & 7 & -3 \end{matrix}$$

$$\frac{x_1}{20-21} = \frac{x_2}{-49+50} = \frac{x_3}{-30+28}$$

$\therefore$ Eigen vector $X_2 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

case 3 : When $\lambda = 2$ put in equ (1)

$$\begin{bmatrix} 11-2 & -4 & -7 \\ 7 & -2-2 & -5 \\ 10 & -4 & -6-2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 9 & -4 & -7 \\ 7 & -4 & -5 \\ 10 & -4 & -8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$(\lambda - 1)(\lambda - 2) = 0$$

$$\lambda = 1, 2$$

$\therefore$ Eigen values 0, 1 and 2

Step 3 : To find eigen vectors

$$(A - \lambda I)X = 0$$

$$\begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 11-\lambda & -4 & -7 \\ 7 & -2-\lambda & -5 \\ 10 & -4 & -6-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

case 1 : when $\lambda = 0$ put in equ (1)

$$\begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$11x_1 - 4x_2 - 7x_3 = 0 \longrightarrow (1)$$

$$7x_1 - 2x_2 - 5x_3 = 0 \longrightarrow (2)$$

$$10x_1 - 4x_2 - 6x_3 = 0 \longrightarrow (3)$$

consider (i) & ii) solve using cross multiplication

$$\begin{array}{cccc} -4 & -7 & 11 & -4 \\ -2 & -5 & 7 & -2 \end{array}$$

$$\frac{x_1}{20 - 14} = \frac{x_2}{-49 + 55} = \frac{x_3}{-22 + 28}$$

$9x_1 - 4x_2 - 7x_3 = 0 \longrightarrow (1)$

$7x_1 - 4x_2 - 5x_3 = 0 \longrightarrow (2)$

$10x_1 - 4x_2 - 8x_3 = 0 \longrightarrow (3)$

consider (i) & ii) solve using cross multiplication

$$\begin{matrix} -4 & -7 & 9 & -4 \\ -4 & -5 & 7 & -4 \end{matrix}$$

$$\frac{x_1}{20-28} = \frac{x_2}{-49+45} = \frac{x_3}{-36+28}$$

$\therefore$ Eigen vector $= \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$

RESULT:

Eigen values 0, 1, 2

Eigen vectors $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$

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3. Using Calyley-Hamilton theorem verify $A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix}$ and find A^{-1}

Sol:

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix}$$

Characteristic equation. $\lambda^3 - C_1\lambda^2 + C_2\lambda - C_3 = 0$

C_1 = Sum of diagonal elements

$= 1 + 2 + 2 = 5$

C_2 = sum of minors of diagonal element

$= (4-0) + (2-0) + (2-0) = 4 + 2 + 2 = 8$

$C_3 = |A| = 1(4-0) - 0(4-0) + (-2)(0-0) = 4$

$\therefore$ Equation $\lambda^3 - 5\lambda^2 + 8\lambda - 4 = 0$

using by caley - Hamilton theorem

$$A^3 - 5A^2 + 8A - 4I = 0$$

$$A^2 = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -6 \\ 6 & 4 & 12 \\ 0 & 0 & 4 \end{bmatrix}$$

$$A^3 = A^2 \times A$$

$$= \begin{bmatrix} 1 & 0 & -6 \\ 6 & 4 & 12 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -14 \\ 14 & 8 & 28 \\ 0 & 0 & 8 \end{bmatrix}$$

$A^3 - 5A^2 + 8A - 4I$

$$= \begin{bmatrix} 1 & 0 & -14 \\ 14 & 8 & 28 \\ 0 & 0 & 8 \end{bmatrix} - \begin{bmatrix} 5 & 0 & -30 \\ 30 & 20 & 60 \\ 0 & 0 & 20 \end{bmatrix} + \begin{bmatrix} 8 & 0 & -16 \\ 16 & 16 & 32 \\ 0 & 0 & 16 \end{bmatrix} \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0 \quad \text{Hence proved}$$

To find A^{-1}

$$A^3 - 5A^2 + 8A - 4I = 0$$

$\div A$

$$4A^{-1} = A^2 - 5A + 8I$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1/2 & 2 \\ 0 & 0 & 1/2 \end{bmatrix}$$

Step 3: To find eigen vectors $(A-\lambda I)X=0$

$$\left[\begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}\right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 7-\lambda & -2 & 0 \\ -2 & 6-\lambda & -2 \\ 0 & -2 & 5-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

case 1: when $\lambda = 3$ put in (1)

$$\begin{bmatrix} 7-3 & -2 & 0 \\ -2 & 6-3 & -2 \\ 0 & -2 & 5-3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -2 & 0 \\ -2 & 3 & -2 \\ 0 & -2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-4x_1 - 2x_2 = 0 \longrightarrow (1)$$

$$-2x_1 + 3x_2 - 2x_3 = 0 \longrightarrow (2)$$

$$0x_1 - 2x_2 + 2x_3 = 0 \longrightarrow (3)$$

From (ii) & (iii) solve using cross multiplication

$$\begin{matrix} 3 & -2 & -2 & 3 \\ -2 & 2 & 0 & -2 \end{matrix}$$

$$\frac{x_1}{6-4} = \frac{x_2}{0+4} = \frac{x_3}{4-0}$$

$$\frac{x_1}{2} = \frac{x_2}{4} = \frac{x_3}{4}$$

$$\frac{x_1}{1} = \frac{x_2}{2} = \frac{x_3}{2}$$

$\therefore$ Eigen vector $X_1 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$

case 2 : when $\lambda = 6$ put in ,

$$\begin{bmatrix} 7-6 & -2 & 0 \\ -2 & 6-6 & -2 \\ 0 & -2 & 5-6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 0 \\ -2 & 0 & -2 \\ 0 & -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 - 2x_2 + 0x_3 = 0 \longrightarrow (1)$$
$$-2x_1 + 0x_2 - 2x_3 = 0 \longrightarrow (2)$$
$$0x_2 - 2x_2 - x_3 = 0 \longrightarrow (3)$$

From i) & ii) solve by cross multiplication

$$\begin{matrix} -2 & & 0 & & 1 & & -2 \\ & \times & & \times & & \times & \\ 0 & & -2 & & -2 & & 0 \end{matrix}$$

$$\frac{x_1}{4-0} = \frac{x_2}{0+2} = \frac{x_3}{0-4}$$

$$\frac{x_1}{4} = \frac{x_2}{2} = \frac{x_3}{-4}$$

$\therefore$ Eigen vector $X_2 = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$

case 3 : when $\lambda = 9$ put in (1)

$$\begin{bmatrix} 7-9 & -2 & 0 \\ -2 & 6-9 & -2 \\ 0 & -2 & 5-9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$